LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.SC., B.C.A. DEGREE EXAMINATION – **COMP.SCI., & COMP. APP., PHY.,STAT., AND CHEM.**

THIRD SEMESTER – NOVEMBER 2018

MT 3206 – APPLIED MATHEMATICS

Date: 26-10-2018   Dept. No. [   ]   Max. : 100 Marks
Time: 01:00-04:00

SECTION A

Answer **ALL** questions: **(10 x 2 = 20)**

1. If the Revenue function $R^{'}(x) = 12 - 8x + x^2$. Determine the total revenue and demand function.

2. Integrate $x^{\frac{3}{2}} + x^{\frac{1}{4}}$ with respect to x.

3. Find the maximum value of the directional derivative of $\varphi = 2x^2 + 3y^2 + 5z^2$ at the point $(1, 1 - 4)$.

4. If $\vec{F} = t^3\vec{i} + t^2\vec{j} + (3t + 1)\vec{k}$. Find $\frac{d^2\vec{F}}{dt^2}$.

5. Show that $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is solenoidal.

6. Identify the degree of the differential equation $y = \sqrt{x}\frac{dy}{dx} + \frac{k}{\frac{dy}{dx}}$.

7. State Shifting property in Laplace Transforms.

8. Prove that $L\{1\} = \frac{1}{s}$ if $s > 0$.

9. Define Linear Programming problem.

10. Define Sperman's rank correlation coefficient.

SECTION B

Answer any **FIVE** questions: **(5 x 8 = 40)**

11. If supply function is $y = 2x^2 + 4$, given $x_0 = 2$, $p_0 = 12$. Find the producer surplus.

12. A particle moves along a curve whose position vector at any time t is given by $\vec{r} = t^3\vec{i} + (t^2 - 1)\vec{j} + 4t\vec{k}$. Find the velocity and acceleration at time $t = 1$.

13. Prove that $div\ r^n\vec{r} = (n + 3)r^n$.

14. Find the center of gravity of a semi-circular lamina defined by $x^2 + y^2 \leq 4; x \geq 0$.

15. A $12v$ battery is connected to a simple series circuit in which the inductance is $\frac{H}{2}$ and resistance is 10Ω. Determine the current i of $i(0) = 0$.

16. Evaluate $\int_0^{\infty} e^{-2t} \sin 3t\ dt$.

17. Find $L^{-1}\left[\frac{1}{(s+1)(s+3)}\right]$.

18. From the following data calculate the coefficient of correlation.

X	1	2	3	4	5
Y	10	20	30	40	50

SECTION C

Answer any **TWO** questions: **(2 x 20 = 40)**

19. (a) The quantity sold and the corresponding price, under monopoly is determined by the demand function $y = 16 - x^2$ and the marginal cost function $y^{'} = 6 + x$ in such a way as to maximize the profit. Determine the corresponding consumer surplus.

(b) Evaluate $\int_0^3 \int_1^2 xy(x+y)\,dydx$ and $\int_1^2 \int_0^3 xy(x+y)\,dxdy$ and show that they are equal.

(8+12)

20. (a) If $f(x) = x(x^2-4)^2, 1 \le x \le 3$. Prove that $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(b) Evaluate $\nabla(\log r)$ where $r = |\vec{r}|, \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

(12+8)

21. (a) Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ given that $y(0) = 2, y'(0) = 2$ when $t = 0$.

(b) A fossilized bone is found to contain $\frac{1}{1000}$ to the original amount of ^{14}C (Carbon − 14). Determine the age of the fossil.

(12+8)

22. Calculate the standard deviation, coefficient of variation and variance for the following data:

No. of students	5	15	25	35	45	55
Marks	10	20	30	50	40	30

(20)

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